Overview of Credit Valuation Adjustments

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Summary

- Banks adjust theoretical value of derivatives due to presence of
  - ◇ residual credit risk,
  - ◇ capital requirements.
- Valuing these adjustments on new business is a corporate finance problem, which in practice is proxied by a derivative pricing problem.
- It affects buy-side firms in various ways...
- ... but also provides a potentially lucrative investment opportunity.

Outline

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Why is XVA relevant

- XVA is an umbrella term for several credit and capital-related adjustments to the theoretical price of derivatives.
- Since Basel III, banks must do it and they also have to actively manage market and credit risk on those.
- The adjustments result from the fact that derivatives are bought and sold by real firms (with balance sheet).
- The subject is relevant for the buy side:
  - ◇ Make sense sell-side pricing
  - ◇ Negotiate ISDAs, including Initial Margin (IM)
  - ◇ Understand motivation behind the limit setting process by the banks
  - ◇ Consider investment opportunities in XVA and to relieve banks' capital by mitigating XVA

Outline

What does "X" stand for

- CVA: credit value adjustment:
  - haircut for credit riskiness of the counterparty.
- DVA: debit value adjustment:
  - "CVA" on yourself from the counterparty's point of view.
- FVA: funding value adjustment:
  - cost of hedging an uncollateralized derivative with a collateralized one.
- KVA: capital (requirement) value adjustment:
  - transfer or marginal capital requirement to counterparty.
- MVA: margin value adjustment:
  - cost of funding margins.

Creation of XVA

- Consider an unlevered firm with balance sheet equation

$$A_0 = E_0$$

- Changes in asset value generates PNL

$$A_t = E_0 + PNL_t,$$

but default is not possible since there is no liability.

- If the firm trades a derivative then it *creates* marginal asset and/or liability.
- E.g. Interest rate swap is a package referencing *existing* fixed and a floating bond priced at par.

$$A_0 + FloatingBond_0 \leftrightarrow E_0 + FixedBond_0$$

- No equality in the above even if $FloatingBond_0 = FixedBond_0$, due to *created* XVA.

Basic structural model ("Merton")

- In the above example IRS created marginal credit risk.
- As such, pricing such marginal credit risk is a *corporate finance* problem.
- A structural credit model ("Merton") uses option pricing analogy to analyze corporate finance problem ("The pricing of options and corporate liabilities", Black-Scholes, 1973).
- Consider a firm with asset process A_t; the firm's only liability is a zero-bond with notional N maturing at time T.
  - ◇ (We start with this trivial case and then generalize to a time "continuum" of bonds with stochastic notionals.)
- What is the value of this bond B_t and of equity E_t?

DVA

- Assume that rates are zero.
- Time-T payoff to the bond holders is

$$\min(N, A_T) = N - max(N - A_T, 0) = N - (N - A_T)^+$$

- The second term is a "put" payoff, representing loss in case of default.
- The value of bond is

$$B_t = N - \mathbb{E}^Q(N - A_T)^+ = N - Put(t, A_t, N) = N - DVA$$

- DVA is the bond's haircut due to credit risk.

Capital,Balance Sheet and PNL

- Payoff to equity holders is "call" on assets, therefore the value is

$$E_t = \mathbb{E}^Q (A_T - N)^+$$

- The balance sheet equation is

$$A_t = E_t + N - DVA_t$$
$$DVA_t + A_t = E_t + N$$

which is call-put parity.

- PNL is still driven by assets, but haircut by change in DVA

$$\mathrm{d}DVA_t + \mathrm{d}A_t = \mathrm{d}E_t,$$

and note that $\partial DVA_t / \partial A_t < 0$, as it is a "put".

Derivatives as marginal dynamic assets/liabilities

- The above case is generalized to derivatives by making N_T stochastic (a "contingent" claim).
- The non-stochastic case formulas can be considered conditioned on one terminal value of N_T.
- Unconditional value can be obtained by taking expectations w.r.t. N_T

$$DVA = \int \mathbb{E}^{Q|N_T}(A_T - N_T)^+ \mathrm{d}P(N_T)$$

- The key take away that every time bank does a derivative trade a new DVA is *created*.
- It is assumed by the investor (counterparty) and *cannot be hedged*; but it can be sold physically or synthetically (via CDS).

(Original) Wrong Way Risk

- Wrong/Right way risk in mainstream XVA means positive/negative correlation of credit exposure and credit event.
- The structural view such risk is almost always present, since DVA is driven by

$$A_T - N_T,$$

  i.e. exposure (N_T) explicitly drives default
- This becomes relevant especially for buy-side if A_T and N_T are correlated.

CVA

- CVA is the DVA viewed from the investor's perspective (i.e. the one who buys the bond).
- When derivative is such that it can be both an asset and a liability, then the deal will have both CVA and DVA adjustments.
- In case of 2 counterparties the solution is intuitive.
  - ◇ Assume derivative value process is M_t.
  - ◇ Then for a given counterparty (1) it is either asset with value$A_t^{(1)} + M_t^+$or a liability with value $-M_t^-$backed by asset $A_t^{(1)}$.
  - ◇ $-M_t^-$must be haircut by DVA.
  - ◇ M_t^+ must be haircut by CVA which is driven by assets $A_t^{(2)}$ of the other counterparty.

Multiperiod case

- Multiperiod case is obtained by considering dynamic liabilities with relevant maturities.
- It is important that default can only happen on dates when payments are due!
- This is equivalent to conditioning on default time
- The full DVA is then given by yet another outer integral

$$DVA = \int \int \mathbb{E}^{Q|N_T}(A_T - N_T)^+ \mathrm{d}P(N_T)\mathrm{d}T$$

Marginal pricing vs marking to market

- The key idea in pricing derivatives is replication in a complete and arbitrage-free market
  - All risk are tradeable
  - Martingale measure exists
  - No new risks are created
- Therefore "initial pricing" is same as marking to market, as the value of a derivative is same as the value of the replicating portfolio.
- In CVA/DVA, only reference contingent claim exists, while credit risk is created.
- Once it is created, it can be marked to market (theoretically), assuming that it can be sold, at least via a CDS.
- This is the key theoretical difference!
  - Lot's of research still equates mark to market of CVA with initial pricing.

Outline

Problems with Merton model

- XVA was originally done not by corporate finance people, but by credit derivative ones.
- They do not like Merton model, as it is hard to make work in practice.
    - ◇ Requires assumption about dynamics of unobserved assets.
    - ◇ Relies on the balance sheet data.
    - ◇ Hard to calibrate to observed credit spreads.
- Machinery of reduced form model was already in place by the time XVA became relevant.

Reduced form model

- Assume default probability is given endogenously as a "survival probability" function $S(t)$:

$$S(t) > 0,$$
$$S(0) = 1,$$
$$S'(t) \leq 0.$$

- Then the time-t present value of a random payoff X_u on default at time u will be given by

$$V_t = \mathbb{E}\left(\int_t^T X_u e^{-r(u-t)}\left[-\mathrm{d}S_u\right]\right)$$
$$\approx \sum \mathbb{E}\left(X_i\right) e^{-r(u_i-t)}\left(S(t_i) - S(t_{i-1})\right).$$

CVA as CCDS (1)

- In case of a bond $X_u = (1 - R)N$, where N is bond's notional and R is recovery rate.
- V_t is then the value of the "protection" leg of a CDS: we are paid notional of bond by the protection seller and give them the defaulted bond, on which they recover R.
- Assume we have a derivative trade with the counterparty, and the trade's mark to market process from our perspective is M_u.
- Also assume that M_u is collateralized and the collateral process is C_u.
- If $M_u - C_u > 0$ then we are owed money and we will lose $(1 - R)(M_u - C_u)^+$ should the counterparty default at time u.

CVA as CCDS (2)

- So it seems that if we make $X_u = (1-R)M_u^+$, then

$$CVA = \mathbb{E}\left(\int_t^T (1-R)\,(M_u - C_u)^+\, e^{-r(u-t)}\,[-\mathrm{d}S_u]\right)$$
$$\approx (1-R)\sum \mathbb{E}\,(M_{u_i} - C_{u_i})^+\, e^{-r(u_i-t)}\,(S(t_i) - S(t_{i-1}))\,.$$

  is exactly the haircut to the credit-risk free to the value of the derivative.
- It, kind of, is, but it is called "Contingent CDS" (contingent on M_u), but it is not exactly CVA.
- CCDS is, effectively, a CDS with time dependent notional, know as "expected exposure profile" $\mathbb{E}\,(M_u - C_u)^+$.
- In reality, M_u is the netted value of portfolio of derivatives between two counterparties.

Reduced form model limitations

- Existence of $S(t)$ requires existence of CDSs on the counterparty, i.e. existence of tradeable bonds on the counterparty.
- The case of unlevered counterparty cannot be handled at all.
- Marginal nature of liability brought by a derivative is ignored; marginal cost is assumed equal to average cost, embedded in $S(t)$.
  - ◇ I.e. no difference between pricing XVA on new business and marking to market.
- Default event is most often considered independent from exposure; no Original WWR.
- WWR is sometimes introduced when ignoring it is too unrealistic (e.g. in computing CVA on EM FX trades done with EM banks).

How and why it mostly works

- Thus, a corporate finance problem is proxied by a derivative pricing problem.
  - ◇ Constructing $M_u$is most costly part of calculation and it is more efficient in a "risk-neutral" setting, i.e. using American Monte Carlo.
- But lots of unobserved parameters are proxied in exactly the way it would be done in a corporate finance world.
  - ◇ E.g. cost of debt proxied based on size/geography/industry generics.
- Marginal effects can be ignored if outstanding debt is large, because marginal exposure brought in by derivatives is of the order of 10% of the outstanding debt notional (interest payments).

Capital requirements

- Up until Basel II.1/5 dealing with credit charges was the matter of prudence, as credit was not part of market risk
- Two schools:
  - ◇ tradeable and hedgeable CVA
  - ◇ CVA as a balance sheet credit charge
- Basel III (post 2008) brought in both components, but not in always consistent way
  - ◇ Counterparty Credit Risk (CRR) Charge

    ...what you would have on a load
  - ◇ CVA VAR ("Market Risk") Charge

    ...to motivate actively hedge CVA

Risk-neutral vs historical measure valuation

- Risk neutral valuation treats CVA as CCDS
- Requires a complex hybrid model calibrated!
  - ◇ Exposure profile on an single IRS is like a strip of off-the-money swaptions.
- Difficult to construct consistently (if possible at all), depends on many unobserved correlations.
  - ◇ ... but allows for fast generation of M_u
- Historical measure model is easier to construct and estimate.
- Slow to generate M_u, especially on exotics, full revaluation is needed.
- Predictive power is small
  - ◇ ...especially for dependence structure and trends.

CVA VAR charge

- Treating CVA as a derivative with present value

$$(1-R)\sum \mathbb{E}\left(M_{u_i}-C_{u_i}\right)^{+} e^{-r(u_i-t)}\left(S(t_i)-S(t_{i-1})\right)$$

- Market risk is due to M_u, $C_u$and $S(t)$.
  - ◇ Basel III required that the only source of market risk was $S(t)$!
- Initially captured by VAR.
- FRTB replaces VAR with Expected Shortfall, effectively an "average VAR" for the set of levels.
- $M_u$could be modeled in historical or risk-neutral way.

Credit limit setting

- As opposed to computing expected exposure $\mathbb{E}(M_u)^+$, credit limits are typically set by constructing a "peak" exposure.
  - ◇ For future times construct distributions of M_u.
  - ◇ Compute a curve of given quantiles of the distribution (e.g. 95%).
  - ◇ Take the maximum.
- For this calculation M_u need to be in "historical" measure
- No efficient method to compute distributions
- Ability to capture (much) more market risk factors in a consistent way.

CCR charge

- Initially invented for the portfolio of "loans"
- Derivative portfolio proxied as a "loan" with notional "related" to expected exposure $\mathbb{E}\,(M_u - C_u)^+$.
- This way in can be bundled with other loans to the same counterparties.
- Charge is computed by stressing default probability in a "VAR"-like way.
- M_u is to be modeled in a way consistent with the limit.

Initial Margin

- Many banks (mostly in US) got approval to compute CVA charge in "risk-neutral" measure.
- This allowed to reuse existing "front office" derivatives infrastructure and not to invest in a system implementing historical measure modeling.
- IM does require such a model
  - ◇ Similar to VAR, but
  - ◇ Possibly longer term
- Naive "historical simulation" will not work for "mean reverting" time series.

Outline

XVA and buy-side firms

Economics of XVA

Practice of XVA

Implications of buy-side counterparties

Investment opportunities and structures

Back to corporate finance

- Bank systems, typically implementing a reduced-form approach are not always fit to deal with buy side.
  - ◇ Buy side firms may not have CDS tradeable

    ...so it needs to be proxied as in corporate finance world.
  - ◇ They may be inclined to take larger marginal exposure,

    ...so marginal effect may not be ignored.
  - ◇ This exposure may come in form of exotic derivatives

    ...so WWR may become significant.
- As such a form of a structural model may be needed (and probably is used).
- Has the bank ever asked the detailed balance sheet?

IM and Residual CVA

- Deals are mostly collateralized, so very little residual CVA driven by $M_u - C_u$
- IM will usually be requested to somewhat mitigate risk over margin period of risk
- It is possible no CVA is going to be computed at all, which will cause underestimation of WWR
  - ◇ ...and it can be large!

Original Wrong Way Risk

- Consider a non-financial firm

A_0	385
DVA	15
Debt	300
Equity	100

- Assume assets are log-normal
- With asset volatility $\sigma = 0.15$, this structure yields 5y CDS level of approx 100bp.

Original Wrong Way Risk

- Assume the firm writes a 5Y ATM put option with max payoff of 10% of its assets
- The risk free value is 5.1 (vs assets of 350 and capital of 100).
- This is a pure liability, so there will be a change in *DVA*
- Mainstream approach would price it of the CDS curve
- Marginal DVA is $5\% \times 5.1 = 0.25$
- Looks very small, so good deal!

Original Wrong Way Risk

- To price with a structural model need to make assumption about the proceeds
  - in cash
  - in more assets (further doubling up)
- In any case valuation needs to be done numerically, as *DVA* is option on assets less liability which is now an option too

Table : Marginal DVAs

Put value	Simple DVA	Cash	Assets
5.1	0.25	3.2	3.7

- Won't drive firm into default, but will dramatically decrease profitability!

Outline

Pricing in capital relief...

- ... is key!
- It is not just a derivative we are pricing, but a derivative issued by a firm!
- Examples:
  - ◇ Contingent CDS
  - ◇ Collateral stand-in

Contingent CDS

- Given a portfolio of derivatives, CCDS pays $(1 - R)(M_u)^+$ in case of a counterparty's default.
- It is, technically, a CDS with notional being expected exposure profile.
- Needs to be collateralized itself.
- Biggest problem to price is dynamic nature of the underlying portfolio, both composition and duration.
- Capital relief needs to be priced in and is the key driver of value pickup.
- Otherwise it is easier just to sell a straight CDS on the reference entity
  - Unless it is the case of investing into an liquid or non-existent CDS

Collateral stand in

- CCDS effectively transfers credit risk on a derivative portfolio as CDS would transfer credit risk in a bond.
- Suppose that an investor would be happy to provide collateral instead of a counterparty that would otherwise not pay collateral.
  - E.g. a commercial company hedging with a bank.
- In doing so, credit risk is "killed at source", as collateral explicitly affects exposure profile

$$\mathbb{E}\left(M_u\right)^+ \Rightarrow \mathbb{E}\left(M_u - C_u\right)^+$$

- This affects both CCR and Market risk charges in *one go*!
- Pricing is essentially of capital relief and portfolio composition is still an issue.