CVA and CCR:

Approaches, Similarities, Contrasts, Implementation

Part 3. Modelling and Valuation

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Outline

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Introduction

CCR vs CVA system scope

Despite aiming at seemingly same goals, the scopes of CCR and CVA systems are quite different.

	CCR	CVA
Generates FMTM	yes	yes
Main output	EE	CVA, DVA, FVA, ...
Application	whole book	actively hedged
Measure	historical or risk neutral	risk neutral
Collateral logic	yes, approximate	yes, part of payoff
Scenarios/deltas	stress test, ad-hoc	deltas, PNL explain
Pre-deal	yes	yes, including deltas
Backtest	regular	ad-hoc
Calibration/estimation	at least quarterly	daily

Introduction

End game: RORC

- Ultimately, the optimal solution is perhaps a unified framework, solving a large scale dynamic programming problem of optimizing return on capital
- Risk neutral/replication valuation to stay
  - as benchmark/quotation mechanism
  - as (almost) exact pricing tool where replication is possible (linear products)
  - as a tool for fast approximations where possible
- The problem is with non-linear products: not clear how to deal with unknown unknowns.

Introduction

One or two different systems in a feedback loop

- CVA/FO system provides a tool to quickly compute hedges and attribute PNL
- Risk system computes the residual capital requirement taking hedging into consideration
- Capital requirement calculations are validated by backtesting and stress testing
- If some businesses do not generate enough return, this must be dealt with ex post
    - in the RORC approach this would be dealt with ex ante by selecting the proper investment/hedging strategy

Introduction

Historical CCR vs risk neutral CVA?

- Regulators do not require CCR model to be historical. This, in principle, opens the door for the banks with strong risk neutral CVA infrastructure to use it also for EPE.
- The issue is that it is not easy to make a risk neutral model a "sufficiently" risky.
- Adding risk premia to the drifts will not make dynamics of the market data similar to the one observed in the past.
- Essential risk factors will be missing:
  - To price CVA on an equity option, one can use Black-Scholes (BS) economy. Only stock needs to be risk neutrally stochastic.
  - For CCR in BS economy, one wants to make not only stock historically stochastic, but also implied vol and evolve both stock and vol in a correlated fashion.

Introduction

CCR: historical vs risk neutral

- Consider BS economy with a single stock
- Risk neutral with historical drift:

$$S_t = S_0 \exp\left(\mu t + \sigma\sqrt{t}X_t\right),$$
$$X_t \sim N(0,1).$$

  Risk neutral drift would be $\mu = r - \sigma^2/2$.
- Historical:

$$S_t = S_0 \exp\left(\mu t + \sigma_0 \exp\left[\mu_\sigma t + \sigma_\sigma\sqrt{t}\left(\rho X_t + \sqrt{1-\rho^2 Y_t}\right)\right]\right),$$
$$X_t, Y_t \sim N(0,1).$$

Introduction: dynamics selection

Complex or easy?

- The typical argument that on the level of a netting set the marginal effects will average out does not hold very well (any more).
- There are small counterparties, for which the averaging out effect would not be sufficient.
- If margines are not Gaussian and are highly correlated, convergence to Gaussian distribution is either slow or may not be happening at all (mathematically).
- For Stressed EPE one will have to stress vols and correlations, adding to the effects described above.
- Anyway, if the goal is to estimate based on the historical time series, then inadequate dynamics will be rejected by the backtest.

Outline

Outline

A realistic CVA payoff

ISDA Master Agreement and CSA

- A unit of CVA calculation is a "netting set", which models the set of transactions covered by an ISDA Master Agreement.
- Master Agreement specifies (among other things)
  1. Termination currency: the currency in which settlement happens. Denote Q_t^i the time-t exchange rate of the $i-$th transaction payment currency into the termination currency (per leg if necessary).
  2. Credit support rules ("CSA"). Denote Q_t^j the time-t exchange rate of the $j-$th collateral asset into the termination currency.
- Note that for pricing purposes,
  1. Valuation currency may differ from termination currency (as valuation currency is dictated by CVA hedging
  2. Netting can be considered at least partially unenforceable.

A realistic CVA payoff

General formula and receivable

A more realistic CVA valuation formula is

$$CVA = \mathbb{E}^Q \left(\int_0^T V_t (1 - R_t^c) \lambda_t^c e^{-\int_0^t (\lambda_u^c + \lambda_u^s) du} D_t dt \right),$$

where V_t is "receivable" on default, which is

$$V_t = Q_t \left(\sum_i Q_t^i MtM_t^i - \sum_j Q_t^j U_t^j C_t^j \right)^+ .$$

Here U_t^j and C_t^j are position in and price of a unit of a collateral asset. In case of cash, $C_t^j = 1$ and Q_t is exchange rate from termination currency into the pricing currency.
Note that V_t is generally discontinuous both due to collateral and coupon payments in MtM_t.

A realistic CVA payoff

Discounting and collateral dynamics

- This valuation approach assumes that cashflows are discounted by the "risk free rate".
- Observe that collateral is, in principle, just another asset in the netting set, only its notional U_t^j is path dependent and is contingent on the rest.
- Collateral cost can therefore be modelled explicitly as part of the implied dynamics of U_t^j, by explicitly charging the collateral accounts.
- This is similar to pricing cash CDOs with complex waterfalls.
- Therefore CSA specific curves are not necessary.

A realistic CVA payoff

Modelling requirements

So the CVA model must provide consistent dynamics of the following:

1. future mark to market (FMTM) values of the trades in the netting set,
2. future values of several FX rates (specific to the set of the netting sets being analyzed),
3. future values of units of the collateral assets, providing credit support to the netting sets,
4. future values of notionals of the collateral assets,
5. counterparty recovery rate (at the time of default).

Of all these, only the last four are true underlying variables. FMTMs are "synthetic" variables, ultimately contingent on even more underlying variables.

Outline

Hybrid model for CVA

Motivating Example

Consider a portfolio of two IRS that has been done at different times in the past and for different maturity. We are effectively pricing a option on the portfolio of those.

- As time has passed, they are almost surely off market: an IR skew model may be necessary
- If their maturities are considerably different, we have an embedded forward starting swap, so a multifactor model may be necessary and the necessity will be increasing as time passes.
- If the swaps are in different currencies an FX model is necessary, such that the marginal IR models have the above properties.
  - FX skew may also be necessary
- Credit contingency in the FX model may be necessary to capture wrong way risk

Hybrid model for CVA

Desirable Features

- Must be modular, support different combinations of factors and different dynamics for a particular factor
- In the minimum: a single currency asset (equity, commodity, , inflation, credit) plus stochastic interest rates.
- In the worst case has to support all those for different currencies plus FX rates
- Long dated evolution for a portfolio: both skew and highly dimensional models may be necessary.
- Support of partial recalibration to optimize sensitivity calculations.
- Necessary at least for benchmarking the simplified version of itself, in case such simplification does produce only tolerable differences.

Hybrid model for CVA

Hub and spoke design

1. Hub: domestic IR.
   - This may be all that is necessary for the simple IRS portfolios.
   - Otherwise necessary as the anchor for discounting
2. Spokes:
   - FX into other currencies
3. Other ends of spokes:
   - other domestic asset classes (equity, credit, commodities, inflation)
   - foreign asset classes, e.g. rates or inflation

Spoke structure allows only restricted dependency modelling (which is hard anyway), but allows to optimize sensitivity calculations.

Hybrid model for CVA

Global vs Netting set calibration

- Typically, low dimensional derivatives models are calibrated for each particular (exotic) trade.
  - This works, because hedging portfolios are "linear" in hedging instruments. So even if hedge notionals are computed using different models, they can be aggregated.
- For CVA one has to calibrate at least for the netting set.
  - For simple netting sets, e.g. containing just a couple of IRS with very similar terms, per netting set calibration is preferable, as long as the pricing model is fast
- Otherwise CVA provides a strong case for global calibration.

Hybrid model for CVA

Rates/credit models: Hull-White (and simple generalizations)

$$dr(t) = (\theta(t) - \lambda(t)r(t))\,dt + \sigma(t)dW(t)$$
$$P(t,T) = A(t,T)e^{-r(t)B(t,T)}$$

- Advantages:
  1. speed and memory efficiency
  2. can also be used for single name credit
  3. may be useful as a building block of a hybrid model, e.g. for some EM currency, especially if coupled with credit
- Issues:
  1. only local calibration possible
  2. calibration not stable for a dynamic portfolio

Hybrid model for CVA

Rates models: multifactor LMM

$$dL_j(t) = \sigma_i(t) L_i(t) dW^{Q^{T_i}}(t)$$

- Worth mentioning because it is almost surely available in the IR quant library.
- Can handle portfolios with wide range of moneyness and maturities.
- Advantages:
  1. flexibility, can be used to benchmark other models
- Issues:
  1. slow and memory hungry
  2. difficult to calibrate, difficult to make work in cross currency setting
  3. takes time to implement, tune and become comfortable with

Hybrid model for CVA

Rates models: Cheyette (Markov HJM with skew)

Consider a general one factor HJM framework

$$df(t,T) = \sigma(t,T)\left(dt\int_t^T \sigma(t,s)ds + dW_t\right).$$

Assuming the separable structure of the volatility

$$\sigma(t,T) = g(t)h(T),$$

where $h(T) > 0$, then

$$f(t,T) = f(0,T) + \frac{h(T)}{h(t)}\left(x(t) + y(t)\frac{\int_t^T h(s)ds}{h(t)}\right),$$

$$r(s) = f(0,t) + x(t),$$

Hybrid model for CVA

Rates models: Cheyette (Markov HJM with skew)

... and

$$P(t,T) = \frac{P(0,T)}{P(0,t)} \exp\left(-\frac{\int_t^T h(s)ds}{h(t)} x(t) - \frac{1}{2}\left[\frac{\int_t^T h(s)ds}{h(t)}\right]^2 y(t)\right),$$

where

$$dx(t) = \left(\frac{h'(t)}{h(t)} x(t) + y(t)\right) dt + g(t)h(t)dW(t)$$
$$dy(t) = \left(g^2(t)h^2(t) + 2\frac{h'(t)}{h(t)} y(t)\right) dt,$$

i.e. bond price is Markov in $x(t)$ and $y(t)$.

Hybrid model for CVA

Rates models: Cheyette (Markov HJM with skew)

Parametrization of $g(t)h(t)$ is the key determinant of the model dynamics

- E.g. local volatility: $g(t)h(t) = a(t) + b(t)x(t)$

Calibration:

1. swaption expansion around ATM or any other point
2. up to 2D PDE. Would be more robust, but ultimate inconsistency with the MC evolution to be used in pricing will have to be handled.

Hybrid model for CVA

Rates models: Cheyette (Markov HJM with skew)

- Advantages
  - still reasonably fast and can introduce the skew
  - a strict extension of HW, which trading is typically familiar with.
- Challenges
  - being one factor, offers very limited ways of decorrelating the curve points,
  - hence not that big added value to HW, unless can be used for netting sets with a reasonable range of maturities.

Hybrid model for CVA

FX/Equity/Commodities: Mixed Volatility Dynamics (MVD)

$$\frac{dS(t)}{S(t)} = f(S,t)\xi(t)dW_S(t) + \ldots,$$

$$\frac{d\xi(t)}{\xi(t)} = \kappa(\log(1) - \log(\xi(t)))dt + \alpha(t)dW_\xi(t)$$

$$dW_S(t)dW_\xi(t) = \rho_{S\xi}(t)dt$$

At each time step T_i, the SV constructs a best fit to Vanilla options maturing at T_{i+1} through the level of $f(T_i)$, vol of vol $\alpha(T_i)$, and correlation $\rho_{S\xi}(T_i)$ The SV process has a fixed mean reversion of κ and the vol of vol incorporates the mixing parameters also chosen externally as:

$$\tilde{\alpha}(T_i) = \beta_1 e^{-\beta_2 T_i}\alpha(T_i)$$

Then LV $f(S, T_i)$ fits the vanillas conditional on the SV parameters.

Hybrid model for CVA

References

- Oliver Brockhaus and Han Lee, Mixed Volatility Dynamics (MVD): Pricing and calibration of long-dated multi-asset products, ICBI Global Derivatives, Paris 2011
- Han Lee and Andrey Chirikhin, CVA, EPE, and Hybrid Derivatives: applications of Mixed Volatility Dynamics (MVD), ICBI Global Derivatives, Barcelona 2012
- William McGhee, The Conditional Integration Approach to Stochastic Volatility Modeling, WBS 8th Fixed Income Conference, Vienna, 2012

Hybrid model for CVA

Inflation: FX analogy

Hybrid rates-inflation dynamics, with Hull-White short rates:

$$dr_N(t) = (\theta_N - \lambda_N r_N(t))\, dt + \sigma_N(t) dW_N(t),$$
$$dr_R(t) = (\theta_R - \lambda_R r_R(t))\, dt + \sigma_R(t) dW_R(t),$$
$$\frac{dI(t)}{I(t)} = (r_N(t) - r_R(t))dt + \xi_I g(I,t) dW_I(t),$$
$$\frac{d\xi_I(t)}{\xi_I(t)} = \mu(\log(1) - \log(\xi_I(t)))dt + \alpha_I(t) dW_{\xi_I}(t)$$

We have used an FX analogy and shown explicit short rate processes for the 'domestic' nominal $r_N(t)$ and 'foreign' real rate $r_R(t)$

The inflation index $I(t)$ is the equivalent of the exchange rate between the nominal and real rates, with its own local vol $g(I,t)$ and stochastic vol $\xi_I(t)$.

All Brownian motions are correlated $\rho_{ij} dt = dW_i(t), dW_j(t)$

Hybrid model for CVA

MVD in practice

- On calibration dates, simulate using large steps
- If intermediate dates are required (e.g. more accurate pricing for continuous barriers) : $t \in (T_i, T_{i+1})$, replace $W^S_{T_{i+1}} - W^S_{T_i}$ by X_t, where with X is a Brownian bridge from $(W^S_{t_i}, t_i)$ to $(W^S_{t_{i+1}}, t_{i+1})$
- Extra care must be taken in the discretization scheme for some products, when using the large time steps and corrections may be required to maintain Martingale conditions
- Correlations must be chosen between Assets $\rho_{S_i S_j}$, between Asset and its SV $\rho_{S_i \sigma_i}$, and finally between the SV processes $\rho_{\sigma_i \sigma_j}$ This can be done through a combination of historical data analysis and calibration when appropriate.

Outline

Collateral dynamics

Terminology

- Calculation dates
- Threshold amount (*TA*)
  - Minimum discrepancy between MtM posted collateral to trigger transfer
- Minimum transfer amount (*MTA*)
  - Minimum amount to be actually transferred
- Independent amount (*IA*)
  - "Add on" to cover variation between calculation dates
- Margin period of risk (*MPR*)
  - Time between default and settlement. No further cashflows happen.

Collateral dynamics

Discontinuity

- The asset itself is just another asset that is modelled by the model
- It is the notional of the collateral asset(s) that needs to be determined to be pugged into the receivable formula
- Existence of collateral calls introduces time discontinuity in the receivable. At calculation date we have (assuming all other variables are continuous)

$$V_{\mathbf{t}^-} = Q_t \left(\sum_i Q_t^i MtM_t^i - \sum_j Q_t^j U_{\mathbf{t}^-}^j C_t^j \right)^+ .$$

- Times need to be handled property if MPR is to be modelled (definitely important for CCR), e.g. $U_{\mathbf{t}^-}^j = U_{t-MPR}^j$. An assumption on the evolution of MtM_t^i also must be made.

Collateral dynamics

Collateral call

- It is tempting to separate modeling the collateral call in the termination currency from the collateral optionality, but this is not possible in general, because one needs to know $U^j_{\mathbf{t}-}$ when deciding how to rebalance them, conditioned on everything else.
- Collateral is actually tracked for a subset of a master agreement set, despite everything is aggregated when computing V_t
- Collateral is called if

$$\sum_i Q^i_t MtM^i_t > TA + MTA - IA$$

- The actual amount of call in the termination currency is

$$\sum_i Q^i_t MtM^i_t - TA$$

for the given collateralization group. The dynamic attribution is an problem of its own that may require American MC.

Outline

Valuation

American Monte Carlo: background

- American Monte Carlo is a technique that was invented to price American/Bermudan derivatives in the MC setting.
- It has to applied when PDE or a tree are hard to implement, for example if dynamics is high dimensional.
- We specifically refer to Longstaff-Schwartz (2001) approach, based on least square projection.
- The convergence properties of the approach for spot valuations were widely researched.
  - See in particular Clemant, Lamberton, Protter (2002) for proof of almost sure convergence.
- The method appears to have become market standard at least for exotics.

Valuation

American Monte Carlo: mechanics

- Assume we are at time t_i, and we have already computed the distribution $MtM_{t_{i+1}}$.
- The relevant point about AMC is that it provides a fast procedure to estimate

$$MtM_{t_i} = \mathbb{E}^Q \left(e^{-r_i(t_{i+1}-t_i)} MtM_{t_{i+1}} \right)$$

- If we were doing that using a binomial tree, in the state k, we would simply put

$$MtM_{t_i,k} = e^{-r_i(t_{i+1}-t_i)} \left[p_{up} MtM_{t_{i+1},k+1} + (1-p_{up}) MtM_{t_{i+1},k} \right],$$

because the tree ignores possible transitions to other than adjacent states.

Valuation

American Monte Carlo: mechanics

- In AMC states belong to different MC paths.
- The argument is therefore, since transitions from all states (all paths) at t_{i+1} to all states at t_i are possible, we can use all t_{i+1}states to estimate the values of all t_i states.
- This is essentially a conditional expectation under the risk neutral measure, given realization of the state variables of the diffusion (or any other measurable variables).
- The idea is therefore to use (some functions of) those state variables as the projection basis of the conditional expectation.
- Typically the functions are first few members of the complete sets of polynomials, which is perfectly consistent with a projection in $\mathcal{L}^2$.

Valuation

American Monte Carlo: convergence

- Clemant, Lamberton, Protter (2002)
- A two stage proof, in the case of pricing of a bermudan:
  - Substitute the exact conditional expectation by a $\mathcal{L}^2$ projection (using the fact that conditional expectation is a projection). Show convergence of the approximate pricing problem to the true pricing problem in $\mathcal{L}^2$.
  - For a given finite $\mathcal{L}^2$ approximation, show that a MC converges almost surely to such an approximation.
- This means that approximation to the bermudan price is as good as the $\mathcal{L}^2$ approximation.
- Pointwise convergence of the FMTM is not proved; but it is not critical for CVA pricing, as we take expectation of the distribution

Valuation

American Monte Carlo: usage for CVA

- To price American security, this "continuation" value would be plugged into the payoff to determine the derivative payoff at the state.
- In CVA there are two options.

1. If we are pricing an American type security, we do apply the payoff first
2. Otherwise we treat the conditional expectation as MtM_{t_i}

- If option is exercised, it is necessary to set the rest of the path to zero for CVA purposes. The approach generalizes to an arbitrary exchange option, which will however require that both option and the exchange payoff are prices during the same rollback.

Outline

Outline

Classes of models: dynamics

Conditioned by product scope and properties of observed time series.

- The goal is to model *inputs* to the pricing models.
- This can be scalars, curves or surfaces.
- It may be required to compute functionals of the diffused values on the fiy (e.g. average value of the stock).
- The key point about selecting dynamics is that it can be easily and quickly estimated from the historical time series (by OLS or MLE at worst).
- The set of risk factors is selected so as to model relevant exposures of the derivatives in the book.

Classes of models: scalars

Typically require dealing with bad residuals

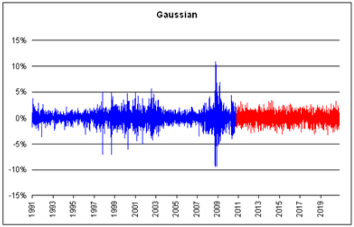
- Typically modelled by a derivative of (Geometric) Brownian Motion (GBM).
- Except perhaps for equity indices, GBM is typically rejected, mostly because of just fat tails, or heteroscedasticity.
- The product scope may require modelling historical stochastic vol.
- The most feasible option for historical stochastic vol is GARCH, as it does not require an extra stochastic factor for the vol:

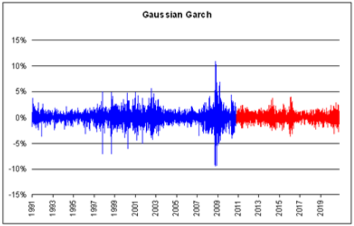
$$\Delta \ln P_t = \mu + \sigma_t \varepsilon_t$$
$$\sigma_t^2 = \sigma_\infty^2 (1 - \beta - \gamma) + \beta \sigma_{t-1}^2 + \gamma \sigma_{t-1}^2 \varepsilon_{t-1}^2$$

Classes of models: GARCH

Example

- It is possible to fit BM (as long as level of significance allows it) to the data which is clearly not BM.
- The out of sample performance will however be very poor
- GARCH provides a reasonable out of sample performance, if in-sample fitting suggests GARCH





Classes of models

EPE Modelling: arbitrage-free curves

- We work with "flat instantaneous forward curves" (easier to rule out arbitrage).
- Select an array of "break" times $\{t_0, t_{1,...,}t_n\}$ define

$$s(t) = \prod_{k=0}^{\lfloor t \rfloor - 1} e^{-h_k(t_{k+1}-t_k)} e^{-h_{\lfloor t \rfloor}(t-t_{\lfloor t \rfloor})},$$

where $\lfloor t \rfloor = \min(i : t - t_k \geq 0)$,is segment selection function and $|\{h_0, ..., h_n\}$ is the array of "forward rates":

$$\frac{d}{dt}[-\ln s(t)] = h_{\lfloor t \rfloor}.$$

Classes of models

EPE Modelling: stochastic Nelson-Siegel model

- The vector of $\{h_0, ..., h_n\}$ is driven by a stochastized version of Nelson-Siegel (NS) model:

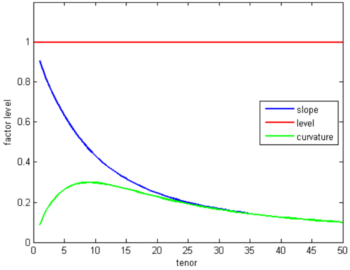
$$
\begin{aligned}
h_{n,t} &= \nu_{1,t}\, f_1(t_n) + \nu_{2,t} f_2(t_n) + \nu_{3,t} f_3(t_n) \\
f_1(\tau) &= 1, \\
f_2(\tau) &= \frac{1 - exp(-\lambda\tau)}{\lambda\tau}, \\
f_3(\tau) &= \frac{1 - exp(-\lambda\tau)}{\lambda\tau} - exp(-\lambda\tau), \\
\nu_{i,t+1} &= \alpha_i + \beta_i \nu_{i,t+1} + \sigma_i \epsilon_{i,t}, \;\; \epsilon_{i,t} \sim N(0,1).
\end{aligned}
$$

Classes of models

EPE Modelling: stochastic Nelson-Siegel model

NS postulates the "smoothed" versions of the typical principal components, which makes it **much handier** for (stressed) scenario analysis.

- NS factors represent the components building up the curve.
- The second and the third factors are related to the Laguerre polynomials.
- Similar shapes are typically extracted by PCA analysis of the individual tenor evolution.



Classes of models

EPE Modelling: NS versus "market standard"

- A more standard approach is to model each h_i directly. For example, if non-negativity is essential then

$$
\begin{aligned}
\nu_{i,t+1} &= \alpha_i + \beta_i \nu_{i,t+1} + \sigma_i \epsilon_{i,t}, \quad \epsilon_{i,t} \sim N(0,1) \\
h_{i,t} &= \exp(\nu_{i,t}) \\
\epsilon_{i,t} & \text{ dependent}
\end{aligned}
$$

- Typically this requires 10-20 tenors to be modelled.
- Correlation matrix of $\epsilon_{i,t}$ is usually PCA'ed to produce 3-5 principle factors.
- The key advantage is that this can fit the initial data better at the expense much heavier evolution.

Classes of models

EPE Modelling: equivalence

Note however that if $\beta_i = \beta$ in NS, then

$$\begin{aligned}
\Delta h_{n,t} &= \sum_{i=1}^{3} \Delta \nu_{i,t}\, f_i(t_n) = \sum_{i=1}^{3} \left(\alpha_i + (\beta_i - 1)\, \nu_{i,t} + \sigma_i \epsilon_{i,t}\right) f_i(t_n) \\
&= \sum_{i=1}^{3} \left(\alpha_i + (\beta - 1)\, \nu_{i,t} + \sigma_i \epsilon_{i,t}\right) f_i(t_n) \\
&= \sum_{i=1}^{3} \alpha_i f_i(t_n) + (\beta - 1) \sum_{i=1}^{3} \nu_{i,t} f_i(t_n) + \sum_{i=1}^{3} f_i(t_n) \sigma_i \epsilon_{i,t} \\
&= A_n + (\beta - 1)\, h_{n,t} + \Sigma \varepsilon_{n,t},
\end{aligned}$$

which is equivalent to "standard" approach with constant β.

Classes of models: curves

Tenor dynamics

- Given the flat forward specification, the array $\{y_0, ..., y_n\}$ is diffused using AR(1) processes, perhaps with GARCH-corrected residuals

$$y_{t+1} = a + by_t + \sigma_t \varepsilon_t$$
$$\sigma_t^2 = \sigma_\infty^2(1 - \beta - \gamma) + \beta\sigma_{t-1}^2 + \gamma\sigma_{t-1}^2\varepsilon_{t-1}^2$$

- y_t can actually the log of the real time series, if non-negativity is essential.
- As opposed to scalar modelling, GARCH does not always help, as returns are fat tailed, but homoscedastic.
- This dynamics appears to be a useful building block, applied to both scalar dynamics and scalar elements of the curve dynamics.

Modelling for CCR: surfaces

Volatilities and correlations

- Key difficulty is to specify dynamics so as to avoid (obvious) arbitrage.
- Diffusing underlying options does not resolve this in a trivial way and introduces high dimensionality.
- Practically three options are possible
  - No diffusion at all; use either constant or local (lookup) volatility/correlation model.
  - Parallel shift of the whole surface. Best works if anchored with GARCH process for historical volatility.
  - Use some surface parametrization (e.g. Sabr) and diffuse parameters. Arbitrage has to be corrected on the fly.

Outline

Evolution models: equity/fx/commodities

- All require scalar evolutions
- Usually will require at least GARCH both to pass backtesting and cover product scope
- Forwards (funding basis/convenience yield) may have to be modelled as a discount-like curve.
- Singe name equity will require a CAPM like model to keep number of risk factors under control.
- Implied vol model for FX not trivial because of quite developed market.
- Biggest challenge is dealing with triangular arbitrage for the options.

Evolution models: rates/inflation

- Need to model:
  - Anchor curves (OIS, or 3m or 6m Libor, depending on currency)
  - Money market basis curves (eg 3s6s)
  - Other basis curves (if necessary), e.g. cross currency
- This is achieved using the flat forward curve model with tenor forward rates diffused independently
- Implied rates vol surface is necessary for non-vanilla options (e.g. Bermudans).
- Implied correlations are necessary for CMS spread products and for quanto products.
- Inflation index will need a scalar model; break even curves need curve model.
- Inflation implied vol model is a challenge to estimate.

Evolution models: credit

- Single name credit /index basis need to be modelled as discount-like curves.
- Need to capture both spread and rating migration dynamics.
- Ideally need a factor model, which is not easy to construct because the process is mean reverting.
- The key factor affecting *EE* is correlated downgrade of the portfolio, not default risk per se!
- Single name curves are best modelled using regime switching approach, where regimes correspond to different "rating" categories.
- Marginal regime switching modelled as a Markov chain.
- Correlated regime switching (in the multiple horizon setting) may or may not be linked to the real equity model.

Evolution models: no arbitrage conditions

- There are two common cases where no-arbitrage conditions have to be enforced: along a curve and across the family of curves.
- The former case is one of all discount like curve and it can be handled by the flat forward parametrization.
- The second case is more difficult, because it is typically formulated not in terms of the curve, but in terms of the value of a tradealbe.
  - E.g. the fact that CDS contracts on the same name with different seniority have ordered credit spreads for all finite tenors does not imply that two credit curves do not cross.
- Same applies for the money market basis curves.
- One approach would. be to introduce the restriction on the level of the flat forward themselves via multiplicative or additive corrections.

Evolution models: no arbitrage conditions

corrections

Inter-curve

Given a curve

$$s(t) = \prod_{k=0}^{\lfloor t \rfloor - 1} e^{-h_k(t_{k+1}-t_k)} e^{-h_{\lfloor t \rfloor}(t - t_{\lfloor t \rfloor})},$$

define

$$h_k = h^{anchor} q_k,$$

$$q_k = \begin{cases} \exp(u_t), & \text{if must be non-negative} \\ 1 - \exp(-\exp(u_t)), & \text{if must be bounded} \end{cases}$$

- Model u_t as a scalar, typically, mean reverting
- Seems trivial and general, but caution: this may result in poorly behaved residuals and make the model useless

Evolution models: dependency

- If we only cared about 1 year horizon (needed for traditional CCR), then correlation approach would be totally adequate.
- A standard approach would be to partition the whole set of factors and PCA their historical correlation matrix.
- A nested PCA may be necessary, e.g. for curve modelling. One would first derive small number (3) factors for each curve and then further PCA "among the curves".
- With introduction of CVA VAR, longer term evolution is necessary for regulatory purposes (not just for internal usage, e.g. for limits)
- Therefore different dependency structures need to be investigated
  - cointegration for unit root processes
  - vector autoregressions

Outline

In the future pricing

Front office, CCR specific

- The general sentiment is that front office pricers are slow. For vanillas this is because of the extra overhead to build market data objects. For exotics, models usually require on-the-fly calibration, which is indeed slow.
- To address the latter issue, CCR model evolves inputs to the pricing algorithm directly, e.g. discount curve, not Libors.
- Vanilla CCR specific pricers can indeed be greatly optimized, but one need to deal with spot valuation differences.
- In the point pricing will be as arbitrage free, as the evolution model is.

In the future pricing

AMC for EPE challenges

- Risk-neutral AMC does not automatically extend to EPE, because EPE MC will evolve not only observables in historical measure, but also model parameters.
  - Essentially CVA (pricing) rollback is the one with the parameters fixed once, during the model calibration.
  - Therefore CVA projections (strictly speaking) cannot be used for EPE, as they are conditioned on the fixed model parameters.
- Without AMC, adding exotics to EPE methodology implies considerably higher hardware requirements, mostly driven by valuation routines.
  - EPE AMC would thus be "MC within MC within MC".
- Given regulators' requirements, EPE pricers need to be sufficiently accurate for the trades to receive IMM treatment.

In the future pricing

AMC for EPE: setup

- Given time slices t_n and t_{n+1}, in CVA AMC we would discount and roll back the vectors of continuation values $V(t_{n+1}, \overline{X}_{t_{n+1}} | \overline{\zeta}^{RN})$,
  - the functions of the projection basis $\overline{X}_{t_{n+1}}$,
  - conditioned on the fixed risk-neutral model parameters $\overline{\zeta}^{RN}$.
- In EPE MC, assume we know $V(t_{n+1}, \overline{X}_{t_{n+1}}, \overline{\zeta}^{RN}_{t_{n+1}} | \zeta^{EPE})$, i.e.
  - (per path) realizations of the values continuation values V,
  - as functions of the "market" variables $\overline{X}_{t_{n+1}}$ and time-t_{n+1} values of the valuation model parameters $\overline{\zeta}^{RN}_{t_{n+1}}$ on the same EPE path,
  - conditioned on the fixed risk-neutral model parameters $\overline{\zeta}^{RN}$.
- We need to project $V(t_{n+1}, \overline{X}_{t_{n+1}}, \overline{\zeta}^{RN}_{t_{n+1}} | \zeta^{EPE})$ risk neutrally onto $\overline{X}_{t_{n+1}}$, conditioned on both $\overline{\zeta}^{RN}_{t_n}$ and ζ^{EPE}.

In the future pricing

AMC for EPE: extension

A possible solution:

1. Put an interpolator on $V\left(t_{n+1}, \overline{X}_{t_{n+1}}, \overline{\zeta}^{RN}_{t_{n+1}} | \zeta^{EPE}\right)$ in terms of $\overline{X}_{t_{n+1}}, \overline{\zeta}^{RN}_{t_{n+1}}$. This can be viewed an equivalent of a "black box" Black-Scholes formula, accepting future spot and implied vol.
2. Perform a single risk-neutral sub-sampling step from t_n into this interpolator, using per-path values of $\overline{\zeta}^{RN}_{t_n}$.

- In the worst case one would sample from each path at t_n, but with smaller number of path per "sub"-sample.
- Bundling paths with close $\overline{\zeta}^{RN}_{t_n}$ allows doing traditional AMC projection within each bundle, thus avoiding direct sub-sampling.
- The key challenge is handling absence of arbitrage in the interpolator.

Outline

Disclaimer

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