CVA and CCR:

Approaches, Similarities, Contrasts, Implementation

Part 2. CVA, DVA, FVA Theory

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Outline

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Review of credit pricing

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Articles

- R. Jarrow, D. Lando, S. Turnbull (1997) A Markov Model for the Term Structure of Credit Risk Spreads.
- D. Lando (1998) On Cox Processes and Credit Risky Securities. Also 1996 dissertation.
- D. Duffie, K. Singleton (1999) Modeling Term Structures of Defaultable Bonds.

Books

- P. Schoenbucher (2003) Credit Derivatives Pricing Models: Models, Pricing and Implementation.
- D. O'Kane (2008) Modelling Single-Name and Multi-Name Credit Derivatives.

Review of credit pricing

Basic notation

τ	Default time, essentially a non-negative number
$1_{\{\tau \leq t\}}$	Default indicator, an increasing $\{0, 1\}$ function of t
τ^C, τ^S	Counterparty and self default times
R	Recovery rate, a random variable on $[0, 1]$
MtM_t	Mark to market of a derivative at time t
	Essentially, a $t-$measurable random variable
$(x)^+$	$max(x, 0)$
$(x)^-$	$min(x, 0)$
EPE_t, ENE_t	Expected positive/negative exposure
	$EPE_t = \mathbb{E}\left(MtM_t^+\right), ENE_t = \mathbb{E}\left(MtM_t^-\right)$
D_t	"Risk free" discount factor at t
S_t	Survival probability by time t, $S_t = \mathbb{E}(1_{\{\tau > t\}})$
N_t	"Notional" (context dependent) at time t

Review of credit pricing

Scope of credit pricing

Credit derivatives deal with compensating the loss due to default after the default happens at time τ. A (cumulative) loss is typically modelled with a jump process L_τ. The elementary credit contingent payoff is simply the increment of L_τ. On the other hand, a fee c is typically payable in a credit derivative, until the default happens.

- Thus the elementary payoffs are:

$$\begin{array}{ll} \text{Protection (floating) payoff:} & \mathrm{d}L_\tau \\ \text{Survival (fee) payoff:} & c\left(N_t - L_t\right) \end{array}$$

- The total time-s value of the protection payoff is

$$P_s = \mathbb{E}^Q\left(\int_s^T D_\tau \mathrm{d}L_\tau\right).$$

Review of credit pricing

Lando integral

- Lando (1996, 1998) and Duffie-Singleton (1999) dealt with pricing of discount protection payoff in continuous time.
- The assumption is that the defaultable claim can be only in two states ("reduced form model") of default/survival, as opposed to migrate to default following dynamics of its credit rating ("full Markov chain" model).
- Assume that default event is modelled as the time of the first jump of a Cox (or general) point process with intensity λ_t.
- Decomposing $L_\tau = \widetilde{L_\tau} 1_{\{\tau \le t\}}$, where $\widetilde{L_\tau}$ is $\tau-$measurable random variable, implies $\mathrm{d}L_\tau = \widetilde{L_\tau} \mathrm{d}1_{\{\tau \le t\}}$.

Review of credit pricing

Lando integral

- The key result (*Lando integral*) is

$$P_s = \mathbb{E}^Q \left(\int_s^T \widetilde{L_\tau} D_\tau \mathrm{d}1_{\{\tau \le t\}} | \mathcal{G}_s \vee \mathcal{H}_s \right)$$
$$= 1_{\{\tau > s\}} \mathbb{E}^Q \left(\int_s^T \widetilde{L_t} \lambda_t e^{-\int_s^t \lambda_u \mathrm{d}u} D_t \mathrm{d}t | \mathcal{G}_s \right).$$

- $\mathcal{G}_s$ is filtration of background information.
- $\mathcal{H}_s$ is filtration of default events.
- Note change in filtrations and time indices of τ for t.
- The inner integral is now really just a simple time integral of paths of the stochastic processes, hence its value is random variable. It is expectation of this variable which is taken.

Review of credit pricing

Survival probability

- Another building block, relevant for CDS pricing (but not much for CVA) was the value of the payoff proportional just to $1_{\{\tau \leq t\}}$(e.g. the fee paid in a swap form)

$$P_s^{fee} = \mathbb{E}^Q \left(\left(\left[1 - 1_{\{\tau \leq T\}} \right] D_T \right) | \mathcal{G}_s \vee \mathcal{H}_s \right)$$
$$= 1_{\{\tau > s\}} \mathbb{E}^Q \left(e^{-\int_s^T \lambda_u du} D_T | \mathcal{G}_s \right).$$

- The term $e^{-\int_s^t \lambda_u du}$ is related to survival probability.S_t. If D_t is not stochastic then

$$\mathbb{E}^Q \left(e^{-\int_s^T \lambda_u du} D_T | \mathcal{G}_s \right) = D_t \mathbb{E}^Q \left(e^{-\int_s^T \lambda_u du} | \mathcal{G}_s \right) = D_T S_T.$$

Review of credit pricing

Recovery rate specification

- Further decomposition of $\widetilde{L}_\tau$ is achieved by introducing a recovery rate R. There are three approaches for the recovery rate specification (originally used in the context of pricing credit risky zero bonds:

Recovery of treasury:	$\widetilde{L}_\tau = (1 - R_\tau) D_T / D_\tau$
Recovery of notional:	$\widetilde{L}_\tau = (1 - R_\tau) N_\tau$
Recovery of "market value":	$\widetilde{L}_\tau = (1 - R_\tau) MtM_\tau$

- The standard approach in credit is to use recovery of notional; this specification is normally used in pricing CDS, yielding

$$P_s = \mathbb{E}^Q \left(\int_s^T (1 - R_t) N_t \lambda_t e^{-\int_s^t \lambda_u \mathrm{d}u} D_t \mathrm{d}t \,|\, \mathcal{G}_s \right).$$

Review of credit pricing

Recovery rate specification

- Recovery of notional is most tractable, as it takes discounting out of pricing. In particular for the risky zero bond we have

$$Z_T = \mathbb{E}\left(R_\tau \frac{D_T}{D_\tau} D_\tau 1_{\{\tau \leq T\}} + D_T 1_{\{\tau > T\}}\right)$$
$$= \mathbb{E}\left(\left([1-(1-R_\tau)]\, D_T 1_{\{\tau \leq T\}} + D_T 1_{\{\tau > T\}}\right)\right.$$
$$= D_T - \mathbb{E}(1-R_\tau)D_T = D_T - \text{''}CVA\text{''}.$$

- The initial motivation for recovery of market value was to allow multiple defaults of the same claim. If this is not allowed (only first jump of Cox process is counted), recovery of market value is essentially same as recovery of "stochastic" notional. We will stick with this specification.

Review of credit pricing

Time discretization and proxying

- Except for very few cases where Lando integral can be evaluated analytically, it is approximated in the time axis:

$$\mathbb{E}^Q\left(\int_s^T D_\tau \mathrm{d}L_\tau\right) \approx \mathbb{E}^Q\left(\sum_i D_i \Delta L_i\right) = \sum_i \mathbb{E}^Q\left(D_i \Delta L_i\right).$$

- If rates are independent from loss process:

$$\ldots = \sum_i D_i \mathbb{E}^Q\left(\Delta L_i\right) = \sum_i D_i \Delta \mathbb{E}^Q\left(L_i\right) = \sum_i D_i\left(EL_{i+1} - EL_i\right),$$

where EL_i is *expected loss*.

- This is the most wide spread formula for protection leg in credit, which is often used for proxying.

Review of credit pricing

Generalization to arbitrary cumulative loss functions

- The last formulas imply that to price a credit product it is only necessary to
  - Specify the cumulative loss process, associated with the payoff. This will typically link recovery rates with some "notional" or "market value" dynamics.
  - Specify a model for the joint default indicator dynamics.
- *CDO.* Consider a basket of M names, with notionals N^i, and recoveries R^i. Then $L_t = \min\left(\sum_{j=1}^{M}(1 - R_{\tau_j})N_{\tau_j}1_{\{\tau_j \le t\}} - A, N^{tr}\right)$
- *CDS* is the special case of the above when $M = 1$.
- Default times are modelled explicitly and coupled with a copula.

Review of credit pricing

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Review of credit pricing

First to Default payoff

- In the most exotic (non-digital) *First to Default* the payoff is triggered by the first default in the underlying pool.
- The payoff however is dependent on the identity of the name being first to default. Specifically:

$$L_t = \sum_{j=1}^{M} (1 - R_{\tau_j}) N_{\tau_j} 1_{\{\tau_j = \min_k(\tau_k) \leq t\}}$$

- Note that the above sum really contains just one term, corresponding to the index of the first to default name.
- Apart from that, modelling requirements are same as for any other basket credit product.

Review of credit pricing

Recap and bridge to CVA/DVA

- *CVA/DVA* is about compensating for losses on MtM_t (of a netted set translated into the *termination* currency), contingent on the first default of the issuer and the counterparty.
- The difference of CVA and DVA is default contingency: CVA pays if counterparty is first to default, and DVA pays if the issuer is first to default.
- So it is essentially the protection leg of an exotic First to Default.
- Therefore all we need to is correctly define the (cumulative loss) payoff and respective first to default curve and plug them into the above formulas.

Outline

CVA/DVA Derivation

Literature review

- Articles
  - Vast literature
  - Hull/White (2000),..., Alavian et al (2007), Brigo et al (2008) and sequels, Morini/Prampolini (2010), Fries (2010), Castagna(2011), and finally Hull/White (2011) to name a few.
- Books.
  - Y. Tang, B. Li (2007) Quantitative Analysis, Derivatives Modeling, and Trading Strategies: In the Presence of Counterparty Credit Risk for the Fixed-Income Market.
  - J. Gregory (2009) Counterparty Credit Risk: The New Challenge for Global Financial Markets.
  - G. Cesari et al (2010) Modelling, Pricing, and Hedging Counterparty Credit Exposure: A Technical Guide.
  - many more apparently in the pipeline

CVA/DVA Derivation

Valuation setup and assumptions

- Assume there exists an OTC transaction between two credit risky counterparties.
- The mark to market process of this transaction is MtM_t (from some counterparty's point of view, which we call "self"). Note that this implies that $D_t MtM_t$ is are martingale.
- Also assume that
  - No simultaneous default, i.e. $\Pr(\tau^s = \tau^c) = 0$.
  - The deal terminates before its scheduled maturity upon the first default among the two counterparties.
  - In the latter case the deal instantly settles for cash. Note that wrt the default augmented filtration, first to default time is stopping time, hence $D_t MtM_t$ remains a martingale even if is stops ("settles") at first to default time. Therefore the fact that we just stop MtM_t does not affect it's spot valuation.

CVA/DVA Derivation

Default contingent payoffs

- The above assumptions imply that credit riskiness of the counterparties introduce two extra settlement cashflows.
- It is convenient to further classify this cashflows in terms of their moneyness wrt the defaulting party.

	Default ordering	Moneyness (to self)	Settlement cashflow
1	$\tau^C < \tau^S$	$MtM_\tau > 0$	$MtM_\tau R^c$
2	$\tau^C < \tau^S$	$MtM_\tau < 0$	MtM_τ
3	$\tau^S < \tau^C$	$MtM_\tau > 0$	MtM_τ
4	$\tau^S < \tau^C$	$MtM_\tau < 0$	$MtM_\tau R^s$

- Cases 2 and 3 do not affect spot valuations.
- Cases 1 and 4 cause $D_t MtM_t$ not to be martingale any more.
- We need CVA and DVA to correct for these.

CVA/DVA Derivation

First to default curve

- To plug into Lando integrals, one needs the driving Cox process.
- Sum of two Cox processes is Cox process.
- Intensity of the fist jump of the sum of the Cox processes is the sum of the intensities, see Duffie (2000)
- Thus, payoffs 1 and 4 collectively have the value

$$\mathbb{E}^Q\left(\int_0^T \widetilde{L}_t\left(\lambda_t^c+\lambda_t^s\right)e^{-\int_0^t(\lambda_u^c+\lambda_u^s)\mathrm{d}u}D_t\mathrm{d}t\right)$$

where

$$\widetilde{L}_\tau = MtM^+_{\tau^c}(1-R^c_{\tau^c})1_{\{\tau^c<\tau^s\}} + MtM^-_{\tau^s}(1-R^c_{\tau^s})1_{\{\tau^s<\tau^c\}}.$$

- Note the second term is negative.

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CVA/DVA Derivation

CVA/DVA pricing formulas from Lando integral

Using Fubini's

$$\mathbb{E}^Q\left(\int_0^T \widetilde{L}_t\left(\lambda_t^c+\lambda_t^s\right)e^{-\int_0^t(\lambda_u^c+\lambda_u^s)\mathrm{d}u}D_t\mathrm{d}t\right)$$

$$=\int_0^T \mathbb{E}^Q\left(\widetilde{L}_t\left(\lambda_t^c+\lambda_t^s\right)e^{-\int_0^t(\lambda_u^c+\lambda_u^s)\mathrm{d}u}D_t\right)\mathrm{d}t$$

$$=\int_0^T \mathbb{E}^Q\left(\widetilde{L}_t\left(\lambda_t^c+\lambda_t^s\right)e^{-\int_0^t(\lambda_u^c+\lambda_u^s)\mathrm{d}u}D_t\middle|1_{\{\tau^c\gtrless\tau^s\}}\right)\mathbb{E}^Q\left(1_{\{\tau^c\gtrless\tau^s\}}\right)\mathrm{d}t$$

CVA/DVA Derivation

CVA/DVA pricing formulas from Lando integral

- Observe that

$$\mathbb{E}^Q\left(1_{\{t=\tau^C<\tau^S\}}\right) = \Pr\left(\tau_c \in [t, t+\mathrm{d}t] \middle| \tau_c < \tau_s\right) = \frac{\lambda_t^c}{\lambda_t^c + \lambda_t^s},$$

$$\mathbb{E}^Q\left(1_{\{t=\tau^S<\tau^C\}}\right) = \Pr\left(\tau_s \in [t, t+\mathrm{d}t] \middle| \tau_s < \tau_c\right) = \frac{\lambda_t^s}{\lambda_t^c + \lambda_t^s}$$

- Therefore, plugging this in the last integral and pushing the above probabilities under the left-most expectation yields

$$\ldots = \int_0^T \mathbb{E}^Q\left(MtM_t^+ (1 - R_t^c)\lambda_t^c e^{-\int_0^t (\lambda_u^c + \lambda_u^s)\mathrm{d}u} D_t\right) \mathrm{d}t$$

$$+ \int_0^T \mathbb{E}^Q\left(MtM_t^- (1 - R_t^s)\lambda_t^s e^{-\int_0^t (\lambda_u^c + \lambda_u^s)\mathrm{d}u} D_t\right) \mathrm{d}t$$

CVA/DVA Derivation

CVA/DVA definition

- Finally we can use Fubini's again and define

$$CVA = \mathbb{E}^Q\left(\int_0^T MtM_t^+(1-R_t^c)\lambda_t^c e^{-\int_0^t(\lambda_u^c+\lambda_u^s)\mathrm{d}u} D_t \mathrm{d}t\right)$$
$$DVA = -\mathbb{E}^Q\left(\int_0^T MtM_t^-(1-R_t^s)\lambda_t^s e^{-\int_0^t(\lambda_u^c+\lambda_u^s)\mathrm{d}u} D_t \mathrm{d}t\right)$$

  with the total compensating correction to MtM_t being $CVA - DVA$.
- Observe that these are automatically bilateral formulas.

CVA/DVA Derivation

CVA/DVA discretization

- Because CVA does look more like an FTD (which itself is like a CDS), a better way to discretize the inner intergral is

$$
\begin{aligned}
CVA &= \mathbb{E}^Q\left(\int_0^T MtM_t^+(1-R_t^c)\lambda_t^c e^{-\int_0^t(\lambda_u^c+\lambda_u^s)du} D_t dt\right)\\
&= \mathbb{E}^Q\left(\int_0^T MtM_t^+(1-R_t^c)D_t e^{-\int_0^t \lambda_u^s du}\left[-de^{-\int_0^t(\lambda_u^c+\lambda_u^s)du}\right]\right)\\
&\approx \mathbb{E}^Q\left(\sum_{i=1}^N MtM_{t^i}^+(1-R_{t_i}^c)D_{t_i} e^{-\int_0^{t_i} \lambda_u^s du}\left[-\Delta e^{-\int_0^{t_i}(\lambda_u^c+\lambda_u^s)du}\right]\right)\\
&= \sum_{i=1}^N \mathbb{E}^Q\left(MtM_{t^i}^+(1-R_{t_i}^c)D_{t_i} e^{-\int_0^{t_i} \lambda_u^s du}\left[\begin{array}{c} e^{-\int_0^{t_i}(\lambda_u^c+\lambda_u^s)du} \\ -e^{-\int_0^{t_i+1}(\lambda_u^c+\lambda_u^s)du}\end{array}\right]\right)
\end{aligned}
$$

CVA/DVA Derivation

Uni- vs bilateral correction

- Unless correlation (or, more generally, positive dependence) is high, the bivariate correction is not that big.
- If defaults are independent then the differential is

$$\mathbb{E}^Q\left(\int_0^T MtM_{t^c}^+ (1 - R_{\tau^c}^c)\lambda_t^c e^{-\int_0^t \lambda_u^c du}\left[1 - e^{-\int_0^t \lambda_u^s du}\right] D_t dt\right)$$
$$\simeq \mathbb{E}^Q\left(\int_0^T MtM_t^+ (1 - R_t^c)\lambda_t^c e^{-\int_0^t \lambda_u^c du} D_t dt\right)\left[1 - S^s(T)\right].$$

- The multiplicative correction is $\simeq 1 - S^c(T) \leq 1$, so staying unilateral is conservative.
- See Brigo (2011) for detailed analysis using Marshall-Olkin copula.

CVA/DVA Derivation

Uni- vs bilateral: word of caution

- Not only correlation is important, but also correlation model.
- Consider the case where first to default curves are constructed using a (term structure consistent) Gaussian copula.
- In this case correlation of 1 actually means perfect ordering of default times(which are themselves still stochastic).
- Thus, assuming we are riskier than the counterparty, univariate CVA is not zero, while bivariate CVA is zero, because in this case.

$$\Pr\left(\tau^{s} \geq \tau^{c}\right) = 1.$$

- Correspondingly, the counterparty's bivariate CVA is biggest.
- The bottom line is that credit dependency for CVA would rather be done with dynamic credit model.

Outline

Have we solved anything?

The standard objections

- The general issue with the above valuation formulas is because they utilize the derivative pricing approach.
- The point of derivatives pricing is that a derivative can be replicated by a self financing hedging strategy, i.e. not requiring firm's capital (theoretically).
- It is possible to have of the fair value determination approaches, but they will imply a necessity of capital, so not that good from the derivatives point of view.
- It is often forgotten that existence of implementable hedging strategy is cornerstone of derivative pricing; it is only after assuming absence of arbitrage and market completeness that one can utilize Fundamental Asset Pricing theorems and immediately price by taking expectations.

Have we solved anything?

More subtle issues

- What we have actually solved is pricing of a contingent FTD between two risk free counterparties!
- Such FTD would referece a trade plus two credit risky entities, default of which triggers payoff
  - Note that this is an identity specific FTD
- Such pricing setup is in fact perfectly valid, if we assume that credit market is complete from the risk free counterparites' point of view
- Is this same as fair value adjustment for transactions between two credit risky counterparties?
  - ...for whom the market is obviousely incomlete
- Does the arbitrage argument work at all?

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Have we solved anything?

The paradox

- Consider a credit risky entity
- If it adds an uncollateralized derivative to its investment portfolio, then negative MtM of such derivative constiute a new liability.
  - This is most obvious if we consider an entity that did not have any debt before, so by construction it was not credit risky
  - Exposure to the uncollateralized derivative does make it risky
  - In practice such a situaion would involve proxying of CDS curve
- Even if the entity already had debt and had CDS trading, referencing this debt, this does not immediately imply that new debt would be priced similarly
  - Economically this would depend on the marginal riskiness of the new investment (=derivative in this case)
  - "Correlation" of the derivative to the rest of the entity's assets will be the key determinant

Have we solved anything?

Conclusions

- When we price CVA/DVA we deal with contingent claims on the assets that don't exist yet
  - This is somewhat similar to the differences in pricing options and warrants and this may not be the biggest issue a priory
  - If we do need to proxy a CDS on the counterparty, this does mean that probably we are pricing it wrongly in the economic sense
- Index proxy is very bad sign
  - A comparable proxy still would reference a CDS on a company with existing debt, not necessarily uncollateralized derivatives.
  - So we are really not in the derivatives pricing situation, but in that of pricing an underlying
- Finally, a typical "simplifying" assuption about independence of CDS and exposure in does not hold in principle.

Outline

Settling through a risk free counterparty

A possible solution

- In what follows we will take a macroeconomic view to show how to reduce the problem of bilateral CVA/DVA pricing to that of a unilateral DVA only (from the risky counterparty point of view
- The key requirement will be existence of risk free counterparties that will serve as the "settlement" centers for the credit risky counterparts
- Credit will be obtained by equilibrium among risk free counterparts
- Credit risky counterparts will be price takers
  - Roughly similar ideas in Castagna (2011) and Morini/Prampolini (2010)

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Settling through a risk free counterparty

Macroeconomic argument

- Consider an economy where there are several credit risk free agents and several credit risky agents
- Credit risk is the one of irrevocable loss of lent funds
- We assume that credit risk free agents are net lenders and they are fungible
- The price for credit in this case is an equilibrium price which is equal to expected loss
- Credit risky counterparts do not have direct access to funding other than via credit risk free ones. Thus they are takers of the equilibrium price. From their perspective this is also the arbitrage price, because of the risk free agents are fungible

Settling through a risk free counterparty

Price taking and equilibrium price of credit risk

- Risk free agents mitigate credit risk entirely via diversification
  - Thus price is given by the equilibrium expected loss
  - Credit risky loans are thus given at a haircut and total expected loss equals the total haircut
- Thus on the expectation basis the economy is balanced.
- In this case CDS trading simply means a swap of a credit risky investment for a credit risk free investment between two credit risk free agents
  - So CDS market exists and assumed to be complete for existing debt, credit risk of which is mitigated by diversification

Settling through a risk free counterparty

Contingent CDS

- Suppose a risky agent A approaches a risk free agent for quoting the following structure:
  - long plain derivative with value process V_t, until a risky counterparty B defaults (if before $V's$ maturity)
  - if B defaults then V_t settles for cash for $R^B V_t^+ + V_t^-$
- Thus the payoff on this structure at time of the

$$U_t = V_t 1_{\{\tau > t\}} + \left(R_\tau^B V_\tau^+ + V_\tau^- \right) 1_{\{\tau = t\}}$$

where $B(x, y)$ is money market account accumulated between times x and y.

Settling through a risk free counterparty

Reduction to FTD

By our assumption, from the risk free agent's point of view the market is arbitrage free and complete, therefore he can price such a payoff. If A was risk free then

$$U_0 = V_0 - \mathbb{E}\left(\int_0^T (1 - R^B_\tau) V^+_\tau D(\tau) \mathrm{d}1_{\{\tau < t\}} \right) = V_0 - CVA$$

Since A is risky, there will be CVA against it, so the value process becomes

$$\begin{aligned} U_t &= V_t 1_{\{\tau^B > t\}} 1_{\{\tau^A > t\}} \\ &\quad + \left(R^B_{\tau^B} V^+_{\tau^B} + V^-_{\tau^B} \right) 1_{\{\tau^B = t\}} 1_{\{\tau^A > t\}} \\ &\quad + \left(R^A_{\tau^A} V^-_{\tau^A} + V^+_{\tau^A} \right) 1_{\{\tau^B = t\}} 1_{\{\tau^A > t\}} \end{aligned}$$

Settling through a risk free counterparty

Reduction to FTD

- Thus U_t now contains the payoff of the contingent FTD
- The economic difference is that from A's perspective he just values a contingent CDS
- Since risk free counterparty is the only one that can price it (using arbitrage argument for the B's on-default contingency), he will add up CVA to the whole transaction.
- That CVA correction may or may not be priced in the risk neutral measure, but A has to take its value as risk free counterparty has market power and can transfer price.
- The main conclusion is that only pure DVA pricing (or CVA of the risk free counterparty) is really important.

Outline

Collateral and funding

Problem statement

- To illustrate the effect of collateralization, consider a collateralized transaction between a risky and a risk free counterparty.
- From risky counterparty's point of view, when $MtM_t < 0$, he will have to post collateral
- The total funding cost of such collateral (for one night) is

$$\left(r_t + \lambda_t + f_t\right) MtM_t^-$$

were r_t is risk free rate, λ_r is credit spread and f_t is funding spread, all overnight. Note that this is a negative number (because of MtM_t^-).

- in case of cash collateral $r_t + \lambda_t$ averaged over the market will be reported as "overnight index" rate.

Collateral and funding

Problem statement, cont'd

- Collateral will be posted with the risk free counterparty, and risky counterparty will only be reimbursed

$$-r_t MtM_t^-$$

- This is because risk free counterparty cannot invest in risky assets remaining risk free

Collateral and funding

Problem statement, cont'd

- Conversely, when $MtM_t > 0$, risk free counterparty will post collateral. If collateral is segregated, the risk free counterparty will demand and the risky counterparty will only be able to raise

$$r_t MtM_t^+$$

- If collateral is not segregated then risk free counterparty will demand and the risky counterparty will only be able to raise

$$\left(r_t + \lambda_t + f_t\right) MtM_t^+$$

  assuming he invest in the counterparty similar to itself.
- So in both cases the revenue/cost cancels out

Collateral and funding

Problem statement, cont'd

- Summing up the terms we obtain that the total cashflow for the risky counterparty in case of collateral segregation

$$
\begin{aligned}
&- r_t MtM_t^- + \left(r_t + \lambda_t + f_t\right) MtM_t^- \\
&= \left(\lambda_t + f_t\right) MtM_t^- \\
&= ''DVA'' + ''FVA''
\end{aligned}
$$

- Note that if $f_t = 0$then this is exactly the "cashflow" in DVA which provides the benefit of not paying it.
- Therefore, if "funding spread" considered to be the spread over the risk free rate then "FVA" will include "DVA"

FVA

Two camps: include in price or not

- Economically, FVA is about transferring the funding cost of the funded hedged to the uncollateralized counterparty, in the same way as CVA and DVA transfer credit risk.
  - Vast literature, e.g. Alavian (2011), Palavicini/Perini/Brigo (2011) and upcoming book, Hull/White (2012).
- Include in valuation or not?
- Transfer pricing argument
- How to actually value it?

FVA

Brute force valuation

Palavicini/Perini/Brigo, Funding Valuation Adjustment... (2011).

$$V_t(C,F) = \mathbb{E}_t \left[\Pi(t, T \wedge \tau) + \gamma\left(t, T \wedge \tau; C\right) + \phi\left(t, T \wedge \tau; F\right)\right]$$
$$+ \mathbb{E}_t \left[1_{\{\tau < T\}} D(t,\tau)\theta_\tau\left(C, \varepsilon\right)\right]$$

where

- $\Pi(t, T \wedge \tau)$ is sum discounted pre-default payoffs
- $\gamma\left(t, T \wedge \tau; C\right)$ is collateral margining costs
- $\phi\left(t, T \wedge \tau; F\right)$ are funding and investing costs
- $\theta_\tau\left(C, \varepsilon\right)$ is on default cashflow

FVA

Brute force valuation

- Then FVA is defined as

$$FVA_t\left(C;F\right) = V\left(C,0\right) - V\left(C,F\right)$$

- At least the second component needs to be computed in a recursive scheme. We will touch on this later.
- Therefore one cannot obtain a simple decomposition

$$V_t\left(C,F\right) = V_t\left(C,F\right)^{rf} - CVA + DVA + FVA$$

- Which measure?

Multicurve discounting

Motivation

- Cross currency basis always existed, only recently it has become more pronounced.
- Most important bases are
  - 3s6s: 3 moth vs 6 month tenor swap,
  - CCY swap.
- They actually exist together, because most CCY swaps are traded against USD.
- Standard USD Libor is 3 months
- Other standard Libors are typically 6 months.
- Now also relevant in pricing multicurrency CSA. This is easier achieved in terms of a correction than a curve, actually.

Multicurve discounting

Multicurve setup: effects on risk

- Modern infrastructure will allow all basis curves to be built consistently, starting from either Libor or OIS discount curves.
- The main effect is on risk decomposition.
- In the past, if we priced in a foreign currency we would only have risk to the foreign discount curve.
- Now, if we select GBP as base currency, already in Libor case we will have for a dollar-contingent trade
  - exposure to GBP curve,
  - exposure to 3s6s GBP basis,
  - exposure to USD 3m curve.
- Total risk will still add up (in this case, GBP risks will mostly offset each other).
- Real decomposition depends on the curve internal setup.

Outline

Wrong way risk

Historical motivation and definition

- Dating back roughly to 1998 crisis.
- It was called "wrong way credit exposure" then (Fingner, 2000).
- In the case of a CDS, it is about recognizing that recovery rate is applied to post default value of the notional.
- Originally intended to adjust the PFE or EE curve (used for counterparty limit monitoring) for the fact that the value of the reference *MtM* can abruptly fall at the time of the counterparty's default.
- At the time, counterparty risk modelling and management was a risk function, so most parameters were not market observable.
- Historical observation of either correlation or contingent jump was subject to statistical inference.

Wrong way risk

Current definition

- Credit contingent jump is the true source of the risk, but it is hard to model, calibrate and risk manage.
- One can incorporate contingent jump into emerging market FX, but this will require adjusting drift and recalibrating all existing FX models.
- Therefore typical modern definition of WWR is risk due to positive correlation between the market factors and credit spreads.
- This definition is also supported by the regulators.
- Importantly, one needs to be careful approximating

$$\sum_i \mathbb{E}^Q \left(D_i \Delta L_i \right) \approx \sum_i D_i \mathbb{E}^Q \left(\Delta L_i \right).$$

- It does not generally work for WWR!

Wrong way risk

Mitigation

- As it is hard to calibrate the appropriate model, WWR has to be mostly mitigated via a reserve.
- The reserve may be computed either with respect to the jump parameters of the proper model, or simply via scenario analysis.
- It is not clear where to put the responsibility for the reserve in the reference claim: the relevant trading desk or CVA desk. It is not a problem, in principle, to segregate all WWR management in CVA area, however this will distort the profitability of the underlying desk, if the reference trade is missing the WWR component in valuation.
- CVA team is clearly responsible for the portion of WWR inherit to CVA/DVA.
- Mind possible contagion.

Wrong way risk

Example: Russia 1998

- Before 1997 the non-residents were attracted by high yields (in dollar terms) on the rouble denominated T-bills (GKOs).
- However there was a requirement for the non-residents to hedge their exposure by buying FX forwards from Russian banks.
- After Asian crisis of 1997, oil price fell to around $10 by mid 1998, depleting foreign currency reserves.
- Stock market was falling and yields on GKOs were risking.
- Fair value accounting for banks' GKO portfolios was suspended in early 1998.
- Crisis talks with the IMF to secure funds to support rouble.
- Situation not much different from Greece, only FX forwards were present.

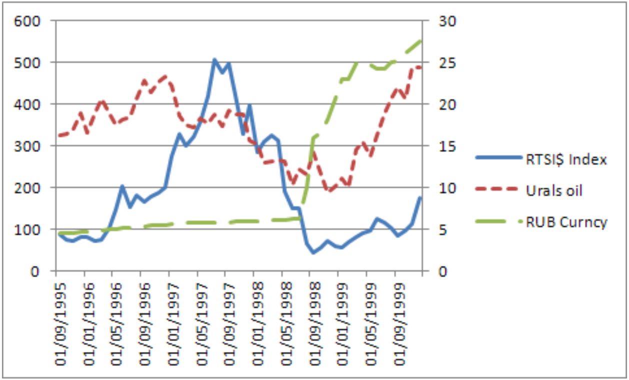
Wrong way risk

Example: Russia 1998

- In the summer of 1998 a feedback loop emerged.
  - MinFin had problems performing GKO actions to roll debt, because of demands for high yield.
  - Pressure on rouble increased both because non-residents were taking money out, and because of FX forward collateral calls.
  - Foreign currency reserves were depleted, so exchange rate could not be maintained.
- In August 1998 rouble was allowed to first devalue by 50%, the upper bound lifted in few days.
- Full devaluation was around 200-300% in a matter of months.
- Major banks, which were counterparties on FX forwards defaulted because GKOs defaulted.

Wrong way risk

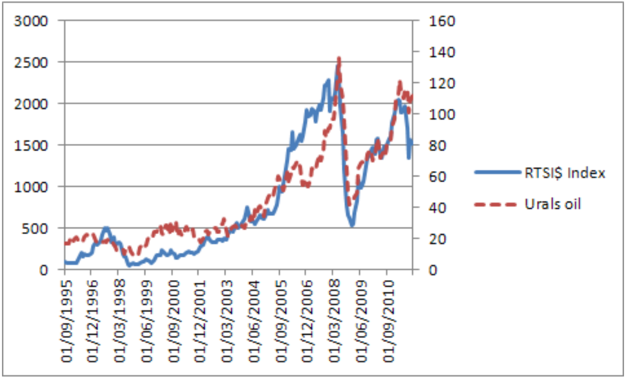
Example: Russia 1998



Source: Bloomberg

Wrong way risk

Example: Russia 1998, happy end



Source: Bloomberg

Outline

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